

# A Study of the Decision of Control Parameters for Adaptive Automatic-Repeat Request Strategy

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## SUMMARY

In many practical transmission systems, transmission error rate of the channel is time-varying. In previous works for controlling the time-varying channel, the state of the channel is estimated and is uniquely determined. Then, the optimum value is chosen as a control parameter for the determined state. However, there is a discrepancy between the actual channel state and estimated state. The optimum control parameter determined for the estimated state does not necessarily provide the best performance presented by some object function in error control such as throughput efficiency. In this study, the expected value of throughput is calculated by using the posterior probability for each state and an algorithm in which the control parameter is selected to maximize the expected performance is proposed. From the viewpoint of the statistical decision theory, it is shown that this selection is the optimum in maximizing the throughput under the Bayes criterion. Using simulation, the proposed method is evaluated and its effectiveness is shown. © 2001 Scripta Technica, Electron Comm Jpn Pt 1, 84(11): 61–70, 2001

**Key words:** Adaptive ARQ; time-varying channel; hidden Markov model; Bayes decision theory.

## 1. Introduction

Because of the rapid progress of information technology, more effective and more reliable communication sys-

tems are in demand. In order to correct errors caused in the channel and improve transmission reliability, FEC and ARQ systems are used. In FEC systems, error correction is carried out at the receiver using error correction codes. In ARQ systems, error is detected using error detection codes and error correction is carried out by sending a signal to the encoder requesting to send the same data. There is a hybrid method (hybrid-ARQ system) in which the above two methods are combined. The ARQ system has been widely used because the equipment is simple and information transmission with high reliability can be achieved even if the channel is considerably noisy.

The performance of error control systems is generally evaluated by throughput rate and reliability (error missing probability). In the channel with a constant error rate (transmission error rate does not vary with time), high reliability and high throughput are obtained by selecting a single error control parameter. However, in some channel, the transmission error rate varies with time (time-varying channel). For example, radio communication, including mobile communication and satellite communication, telephone line and magnetic recording equipment belong to this group. If the present transmission state is known correctly in the time-varying channel, a proper control parameter can be selected at the time. However, practically, it is difficult to know the actual state of the channel.

Conventionally, the present state of the channel is estimated and the proper parameter is chosen based on the estimated state [1, 2]. However, there is a discrepancy between the actual state and estimated state of the channel. Therefore, the control parameter which is best suited for the estimated state does not necessarily optimize the perform-

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ance which is presented with the object function. In some studies, a control parameter is chosen as to improve the average performance for all states without estimating a state [3].

In the algorithm proposed in this study, the expected value of the throughput is calculated based on the posterior probability for each state and a parameter is selected to maximize the expected value of the object function at a time. From the viewpoint of the statistical decision theory, it can be shown that selection is optimal in maximizing the throughput under the Bayes criterion. For the control parameter, such parameters as code length and error correcting region in the hybrid-ARQ could be used. However, in this study, coding rate and selection of code among some codes with different error detection and correction are considered for the control parameter. By comparing the performances of this method and the conventional method using simulation, the effectiveness of this method is demonstrated.

## 2. Preparation

### 2.1. Adaptive error control system

Figure 1 illustrates the adaptive error control system. The data generated at the information source are encoded by an encoder. Then, it is transmitted to a decoder via a forward channel. The channel noise causes transmission errors in received data. When the data are received by the receiver and decoded under the rule of the ARQ or hybrid-ARQ system, the symbol of reception (ack) or retransmission request (nak) is transmitted to the encoder via the feedback channel.

In this study, the state of the forward channel is estimated only from the returned symbol (ack or nak) through the feedback channel. The monitors located at the encoder and decoder estimate the state of the channel, using a series of the returned symbols, and determine the control parameter to be used in the next step. Then, the monitors provide the control parameter to the encoder and decoder,

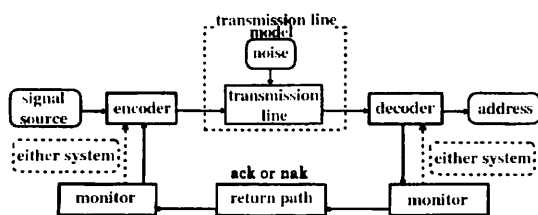


Fig. 1. Adaptive error control system.

and encoding and decoding are carried out depending on the decision.

### 2.2. Channel model

It is assumed that the channel is a binary symmetric channel (BSC). A BSC is determined only by one parameter, bit error rate. In an actual channel, the noise process in general varies with time. However, in a short time, the error rate can be regarded as constant. The most common model is a finite-state Markov model including M-channel states. In this study, a channel is assumed to be modeled by the finite-state Markov model (Fig. 2). Each state represents a BSC with a different bit effort rate. In the actual channel, bit error rate changes slowly compared to the data rate. Therefore, in the model, it can be assumed that state transition occurs only between neighboring states.\*

In a channel, the state transition probability matrix is given as follows:

$$[P_{ij}] = \begin{bmatrix} 1 - P_{01} & P_{01} & 0 & \cdots \\ P_{01} & 1 - P_{01} - P_{12} & P_{12} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & 0 & \cdots \\ & 0 & 0 & \\ & 0 & 0 & \\ * & \vdots & \vdots & \\ & P_{M,M-1} & 1 - P_{M,M-1} & \end{bmatrix} *$$

In this study, the state transition probability matrix of the channel and bit error probability of each channel state are assumed to be known and the present state is assumed to be unknown.

### 2.3. Bayes decision method [6]

The symbols are defined as follows:

- $\theta \in \Theta$ : real state
- $P(\theta)$ : prior probability distribution
- $z$ : observed value
- $d(z)$ : decision function
- $L(\theta, d(z))$ : loss function

where  $z$  is a discrete value.

\* However, the proposed algorithm to be described can be used where this limit is not applied.

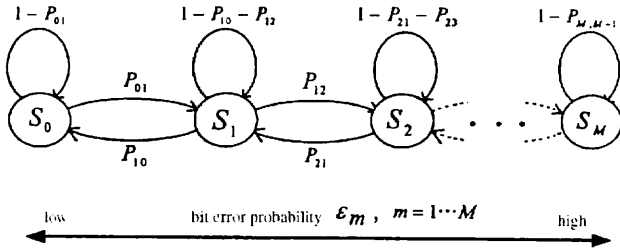


Fig. 2. Channel model.

A risk function is defined by the expected value of a loss function under the appearance probability of  $z$  under state  $\theta$ ,  $P(z|\theta)$ :

$$R(\theta, d) = \sum_z L(\theta, d(z))P(z|\theta) \quad (1)$$

In addition, the expected value of a risk function under the prior probability  $P(\theta)$  at each state is the Bayes risk:

$$BR(d) = \sum_{\theta} \sum_z L(\theta, d(z))P(z|\theta)P(\theta) \quad (2)$$

The condition which minimizes  $BR(d)$  is the Bayes decision.

[Lemma 1]

Optimum Bayes decision is obtained by minimizing the expected value of a loss function under posterior probability. The Bayes decision under observed value  $z$  is shown as follows:

$$\begin{aligned} d_{\text{opt}}(z) &= \arg \min_d BR_z(d) \\ &= \arg \min_d \sum_{\theta} L(\theta, d)P(\theta|z), \end{aligned} \quad (3)$$

where the posterior probability  $P(\theta|z)$  is given as follows:

$$P(\theta|z) = \frac{P(z|\theta)P(\theta)}{\sum_{\theta} P(z|\theta)P(\theta)} \quad (4)$$

### 3. Proposed Method

#### 3.1. Relationship between determination of control parameter and Bayes decision theory

In this study, the return symbol (ack or nak) corresponds to the observed value  $z$  in the statistical decision theory described in Section 2.3. The loss function is the difference between the throughput when the optimum con-

trol parameter is determined for the actual state and when an arbitrary control parameter is determined. The object of this study is to determine the control parameter which minimizes the expected value of loss function, that is, Bayes risk.

In this study,

Selection for control parameter:  $d(z)$

Channel state:  $\theta$

Return symbol:  $z$

are used.

#### 3.2. Estimate of state and decision of control parameter

In the conventional study on the adaptive error control system, the priority is to estimate the present state for deciding the control parameter. However, there is a discrepancy between the actual and estimated states. Therefore, the decision of optimum control parameter for the estimated state does not necessarily provide the optimum performance in an object function for error control such as throughput efficiency.

In this study, the probability of each state is calculated for each stage. The expected value of the loss function for all states and the control parameter are determined to minimize the expected value. To give an example of our method, an algorithm is proposed, where a control parameter is determined providing the maximum expected value of throughput under the condition that the undetectable error probability is less than the given value. Our method can also be used for minimizing the expected undetectable error probability under the condition that throughput is set to be greater than the given value. The control parameter can be the frame length of code or the error correcting region of code in hybrid-ARQ. However, in this study, the coding rate and selection of codes with different error detection/correction capability are chosen as the control parameters.

In this study, the hidden Markov model in which the return symbol (ack or nak) is considered to be the observed symbol is adopted and the sequential algorithm for the hidden Markov model is used [4]. First, from the (appearance) probability of the received observed symbol and state transition probability, the probability of being in each channel state is obtained. Using the probability, the expected value of the throughput when each code is used is obtained. Then, the code which maximizes the expected value of throughput is adopted.

Details are shown below.

(1) First, the initial distribution is given.

For  $i = 1, 2, \dots, M$

$$P_{(0)}(S_i) = \pi_i \quad (5)$$

$P_{(t)}(S_i)$ : the posterior state probability of state  $S_i$  at stage  $t$  after observing the series of return symbols  $fd$  until stage  $t$ .

$\pi_j$ : initial distribution.

(2) If the present time is stage  $t$ , the probability of being in each state at stage  $t + 1$  is calculated using the state transition probability as follows.

For  $j = 1, 2, \dots, M$

$$Q_{(t+1)}(S_j) = \sum_{i=1}^M P_{(t)}(S_i) \cdot P_{ij} \quad (6)$$

$P_{ij}$ : the transition probability from state  $S_i$  to state  $S_j$ .

$Q_{(t+1)}(S_j)$ : posterior state probability of being in state  $S_j$  at stage  $t + 1$  after observing symbols  $fd$  until stage  $t$ .

(3) From the posterior state probability of being in each state at stage  $t + 1$ , the expected value of throughput is obtained.

$$\hat{\eta}_{C(t+1)} = \sum_{j=1}^M Q_{(t+1)}(S_j) \cdot \eta_{j,C} \quad (7)$$

$\eta_{j,C}$ : throughput when code  $C$  is used at state  $S_j$ .

(4) The code which maximizes the expected value of throughput is chosen to be the code used at stage  $t + 1$ .

$$C_{(t+1)} = \arg \max_C \hat{\eta}_{C(t+1)} \quad (8)$$

(5) When the return symbol  $fd_{(t+1)}$  for the codeword sent at stage  $t + 1$  is returned, the probability of being in each state is renewed by the probability that  $fd_{(t+1)}$  is returned when the code  $C_{(t+1)}$  is used at stage  $t + 1$  in the state.

For  $j = 1, 2, \dots, M$

$$P'_{(t+1)}(S_j) = Q_{(t+1)}(S_j) \cdot P_{j,C_{(t+1)}}(fd_{(t+1)}) \quad (9)$$

$P'$ : conditional probability when  $fd$  is observed.

$P_{j,C}(fd_{(t+1)})$ : probability that  $fd_{(t+1)} \in \{ack, nak\}$  is provided when  $C$  is used in state  $S_j$ .

(6) The posterior state probability given observing the series of return symbols  $fd$  until stage  $t + 1$  is calculated by normalizing  $P'$ .

For  $j = 1, 2, \dots, M$

$$P_{(t+1)}(S_j) = \frac{P'_{(t+1)}(S_j)}{\sum_{l=1}^M P'_{(t+1)}(S_l)} \quad (10)$$

Then, go to the state estimation at stage  $t + 2$  [return to procedure (2)].

[Theorem 1] The repetition of control as described above aims at maximizing the expected value of throughput at each stage. This procedure provides the optimum performance under Bayes criterion, that is, Bayes decision is accomplished.

(Proof) It is obvious from Lemma 1.

Refer to Appendix 1 for the above proposed algorithm.

## 4. Simulation

The object of the proposed method is to maximize the expected value of throughput under the condition that the undetectable error probability is kept below the desired value. This procedure is theoretically optimal under the Bayes standard as described above. In order to compare this method with the conventional method, simulation was carried out.

### 4.1. Simulation contents

The achievable throughput by the conventional method and the proposed method are compared in (1). In (2) to (4), simulation is carried out to investigate the throughput characteristic of this proposed method.

(1) Comparison between the study by Rice [1] and the proposed method

The method by Rice is a typical conventional method for determining the control parameter, where the assumption about the channel is similar to the one we used. First, the two methods are compared. Here, the aim is the comparison with a typical adaptive error control system. The study by Rice is considered to be one of the methods whose object is to estimate the state of the channel.\*

In the channel assumed in the study by Rice (corresponding to the case where  $X = 1$  in Fig. 4), Rice's method and the proposed method are compared by simulation.

(2) Comparison between periodic state estimation strategy and optimum state estimation strategy

In order to compare the conventional method that carries out optimum control for one estimated state (which we call the state estimation strategy) and the proposed method using simulation in (3), the following two state estimation strategies are simulated under the same condition as that used for the proposed method. The two following state estimation strategies are methods which generalize the methods for optimizing control for an estimated state.

(a) Periodic state estimation strategy

An observation period with a certain length of stages is set. Then, the monitor observes a series of return symbols received during the period and uniquely determines the channel state in the next stage. The method to select the code which maximizes the throughput for the estimated state\*\* is referred to as the periodic state estimation strategy.

\*Details of Rice's method are given in Appendix 3.

\*\*Almost all the conventional methods belong to this method.



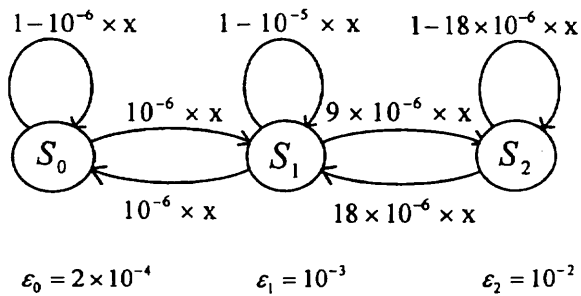


Fig. 4. Assumed channel.

each state is assumed to be as listed in Table 1. The state transition speed depends on factor  $X$  in Fig. 4. As  $X$  increases, the state transition becomes faster. As  $X$  decreases, the state transition becomes slower.

#### 4.2.2. Used codes

The three codes listed in Table 2 are used in simulations (1) to (3). The five codes listed in Table 3 are used in simulation (4). In simulation (4), numerical results are compared for the cases where two codes, three codes, and five codes are used in the proposed method. The codes used in this simulation are listed in Table 4.

The bit error rate and used code for each channel state are determined as described because they are compared with the results obtained by Rice.\*

#### 4.3. Simulation results

(1) Comparison between results by Rice's method and by proposed method

Using the channel assumed in Rice's study (corresponding to the case where  $X = 1$  in Fig. 4), simulations are carried out by both methods. The results are listed in Table 5.

(2) Comparison between the results by the periodic state estimation strategy and optimum state estimation strategy

Figure 5 shows the dependencies of the throughput rate obtained by the conventional method with the two state estimation strategies on the state transition probability whose factor is  $X$  as shown in Fig. 4.

(3) Comparison between the results by the conventional method with the optimum state estimation strategy and the proposed method

\* In Rice's study, simulation was carried out using actual BCH code. In this study, virtual code is used.

Table 1. Each state of channel

State	Bit error possibility
$S_0$	$\epsilon_0 = 2 \times 10^{-4}$
$S_1$	$\epsilon_1 = 10^{-3}$
$S_2$	$\epsilon_2 = 10^{-2}$

Table 2. Used codes (part 1)

Code	Coding Rate $R$	Correctable bit number $t$	Detectable bit number $d$
$C_{120}$	120/127	0	2
$C_{113}$	113/127	1	3
$C_{92}$	92/127	4	6

Table 3. Used codes (part 2)

Code	Coding Rate $R$	Correctable bit number $t$	Detectable bit number $d$
$C_{120}$	120/127	0	2
$C_{113}$	113/127	1	3
$C_{106}$	106/127	2	4
$C_{99}$	99/127	3	5
$C_{92}$	92/127	4	6

Table 4. Used codes (part 3)

	Codes
Two codes used	$C_{120}, C_{92}$
Three codes used	$C_{120}, C_{113}, C_{92}$
Five codes used	$C_{120}, C_{113}, C_{106}, C_{99}, C_{92}$

Table 5. Proposed strategy compared with Rice's strategy

	Throughput
Proposed	0.867844
Conventional (Rice's method)	0.845888

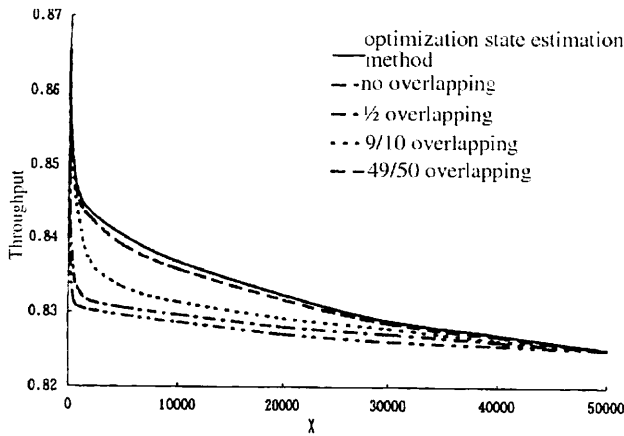


Fig. 5. Optimizing state estimation strategy compared with period strategy.

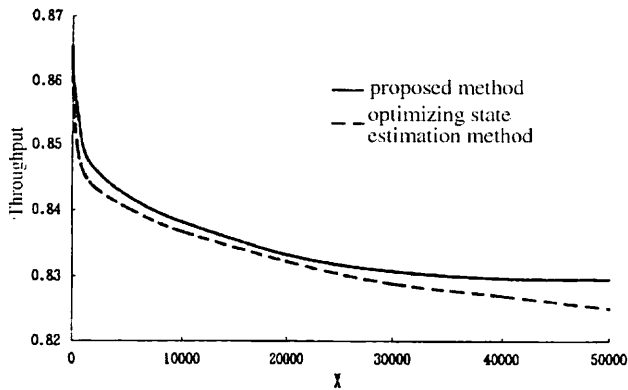


Fig. 6. Proposed strategy compared with optimizing state estimation strategy.

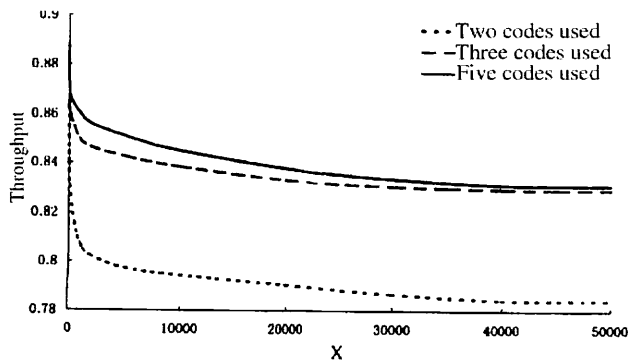


Fig. 7. Comparison of proposed strategy with various numbers of codes.

Figure 6 shows the dependencies of the throughput rate obtained by the proposed method and by the conventional method with the optimum state estimation strategy on  $X$  shown in Fig. 4.

(4) Comparison among the results by the proposed method for various numbers of used codes

Figure 7 shows the dependence of the throughput rate on  $X$ , where  $X$  is illustrated in Fig. 4, obtained by the proposed method.

## 5. Considerations

### 5.1. Discussion of results

(1) Comparison between results obtained by Rice's method and proposed method (Table 5)

The proposed method provides higher throughput than the conventional method.

In Rice's method, the state is uniquely determined when the transition of state is observed by successive evaluation and the optimum code is selected for the state. Then, new successive estimation starts at that point. In Rice's method, the code is successively selected and the observation period is the period between the times when one state transition is observed and when the following state transition is observed. Therefore, this method is one of the periodic state estimation strategies.

(2) Comparison between periodic state estimation strategy and optimum state estimation strategy (Fig. 5)

At any transition speed, the latter provides higher throughput than the former strategy.

In the periodic state estimation strategy as the overlap of the neighboring observation periods increases, the throughput increases. However, even when the overlapping of the period is close to 1 the throughput never exceeds that obtained by the optimum state estimation strategy. The throughput of the periodic state estimation strategy converges as the overlapping increases to the value achieved by the optimum state estimation strategy.

The optimum state evaluation method corresponds to the periodic state estimation strategy in which the period overlapping is 1 and the observation period is infinite. The code selection by the optimum state estimation strategy is the best method from the point that a single state is determined by state estimation and the optimum state estimation strategy theoretically provides the highest throughput of adaptive ARQ schemes whose priority is state estimation.

(3) Comparison between optimum state estimation strategy and proposed method (Fig. 6)

In (2), it was shown that the optimum state estimation strategy always provides higher throughput than the periodic state estimation strategy and that the throughput obtained by the optimum state estimation strategy is highest among those obtained by the state estimation strategies. Here, the proposed method and the conventional method with the optimum state estimation strategy, which is the ideal method among the state estimation strategies, are compared.

The result indicates that the proposed method provides higher throughput than the optimum state evaluation strategy at any state transition speed. When the choice of a state is not clear (when the probability of being in one state and that in another state is almost the same), the conventional method to determine the channel state, such as the optimum state estimation strategy, uniquely determines one of the states. On the other hand, even in the above case (when the probabilities of being in the two states are close to each other), the proposed method selects a code which is adequate to both states and provides a high throughput on average.

That is, the proposed method selects a code to provide the highest expected value of throughput rather than the method in which code is selected for the estimated state.

In the proposed method, as the state transition speed in a channel increases, the throughput decreases and converges to a value, because one code, which is equally adequate to any state, is chosen. On the other hand, in the optimum state estimation strategy, as the state transition speed increases, throughput decreases, because as the state transition speed increases, state estimation becomes more difficult, causing an increase in estimation error.

(4) Comparison among the proposed method for various numbers of code (Fig. 7)

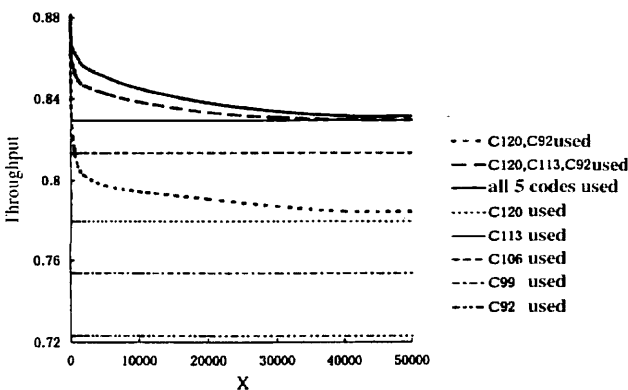


Fig. 8. Convergence property of proposed strategy.

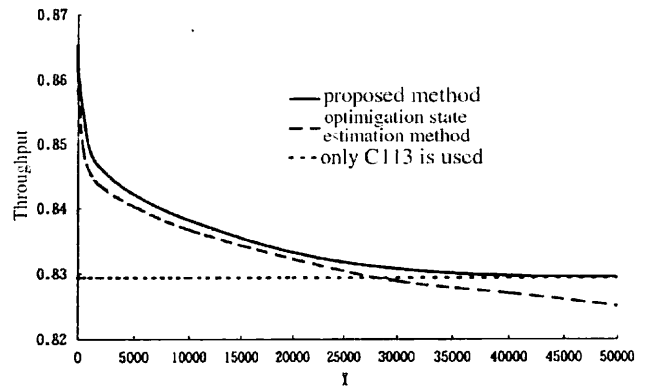


Fig. 9. Throughput property of proposed strategy.

As described above, in the proposed methods, the number of codes need not be the same as the number of states. For example, when the probabilities of being in two states are almost the same, the proposed scheme does not select the code which achieves the best performance in single state, but selects the code which has passable performance in both channels. In the proposed method, finer code selection can be made and higher throughput can be achieved with an increase in the number of usable codes.

In the proposed method, the converged value of throughput obtained when the state transition speed in the channel becomes high depends on what code group is selected as shown in Fig. 8. In the proposed method, when the state transition speed is quite high, the code which is equally adequate to every state of the channel is selected.

At the beginning of this paper, it was mentioned that there is another study which determines single adequate value for each control parameter without estimating the channel state. From the results of this section, it is supposed that the performance of the proposed method would not be inferior to that of the methods using a single optimum code, even when the state transition speed is quite high.

## 6. Conclusions

An optimum algorithm based on the Bayes criterion is proposed to maximize the expected value of throughput under the condition that the undetectable error probability for the time-varying channel is below the desired value.

The proposed method tends to select the best control parameter suited to each state at slow state transition. On the other hand, it tends to select an adequate control parameter which suits every state at high state transition speed. In conventional methods, there are two types of ARQ schemes: the adaptive ARQ method and the ARQ method.



In the former method, the present channel state is estimated and an optimum control parameter is chosen for the estimated state. In the latter method, the channel state is not estimated and a single control parameter adequate for any state is chosen. The proposed method has both advantages of the above two methods and its throughput exceeds those obtained by either method.

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## APPENDIX

### 1. Algorithm for Maximizing Expected Throughput

The algorithm for selecting the code which maximizes the expected value of throughput is as follows:

### 2. Algorithm for Optimum State Estimation Strategy

The algorithm for the optimum state estimation strategy in simulation (3) is shown in Fig. A.2.

```

for  $i = 1, 2, \dots, M$ 
   $P_{(0)}(S_i) = \pi_i$ 
for  $t = 0, 1, 2, \dots$ 
{
  for  $j = 1, 2, \dots, M$ 
     $Q_{(t+1)}(S_j) = \sum_{i=1}^M P_{(t)}(S_i) \cdot P_{ij}$ 
     $C_{(t+1)} = \arg \max_C \{ \sum_{j=1}^M Q_{(t+1)}(S_j) \cdot \eta_{j,C} \}$ 
  for  $j = 1, 2, \dots, M$ 
     $P_{(t+1)}(S_j) = Q_{(t+1)}(S_j) \cdot P_{j,C_{(t+1)}}(fd_{(t+1)})$ 
  for  $j = 1, 2, \dots, M$ 
     $P_{(t+1)}(S_j) = \frac{P_{(t+1)}(S_j)}{\sum_{i=1}^M P_{(t+1)}(S_i)}$ 
}

```

Fig. A.1. Maximizing throughput algorithm.

### 3. Rice's Method [1]

Monitors are placed in the transmitter and receiver stations as shown in Fig. 1 and the monitors receive ack or nak information from the decoder to estimate the channel state. Then, the code adequate to the state is applied to the encoder and decoder.

It is assumed that the channel is a binary symmetrical channel (BSC) and modeled with Markov chain with  $M + 1$  states,  $S_0, S_1, \dots, S_M$ , and the state transition probability,  $P_{ij}$ , as shown in Fig. 2. It is also assumed that the optimum codes for the channel state  $S_0, S_1, \dots, S_M$  are  $C_0, C_1, \dots, C_M$ , respectively. For the code  $C_m$ , sequential test of  $Q_m$  is employed. Sequential test is employed as shown in Fig. A.3.

$S_n$  in Fig. A.3 is given as follows:

```

for  $i = 1, 2, \dots, M$ 
   $P_{(0)}(S_i) = \pi_i$ 
for  $t = 0, 1, 2, \dots$ 
{
  for  $j = 1, 2, \dots, M$ 
     $Q_{(t+1)}(S_j) = \sum_{i=1}^M P_{(t)}(S_i) \cdot P_{ij}$ 
     $C_{(t+1)} = C_{j^*}$ 
    where,  $j^* = \arg \max_j [Q_{(t+1)}(S_j)]$ 
  for  $j = 1, 2, \dots, M$ 
     $P_{(t+1)}(S_j) = Q_{(t+1)}(S_j) \cdot P_{j,C_{(t+1)}}(fd_{(t+1)})$ 
  for  $j = 1, 2, \dots, M$ 
     $P_{(t+1)}(S_j) = \frac{P_{(t+1)}(S_j)}{\sum_{i=1}^M P_{(t+1)}(S_i)}$ 
}

```

Fig. A.2. Algorithm of optimizing state estimation.

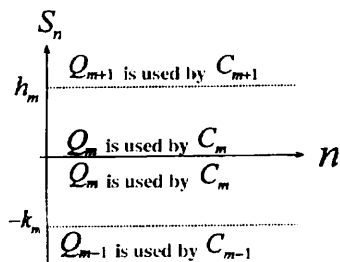


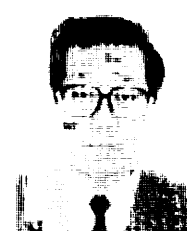
Fig. A.3. Sequential test of channel state.

$$z_i = \begin{cases} -1 & \text{when ack} \\ +b_m & \text{when nak} \end{cases} \quad (\text{A.2})$$

When  $S_n$  exceeds the range between  $h_m$  and  $-k_m$ , a code adequate for the estimated channel state, that is,  $C_{m+1}$  or  $C_{m-1}$ , is selected.

$$S_n = \sum_{i=1}^n z_i, \quad (\text{A.1})$$

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