On the Generalized Viterbi Algorithm using Likelihood Ratio Testing

Toshihiro NIINOMI
Kanagawa Institute of Tech.,
Atsugi-shi, Kanagawa, Japan.
e-mail: niinomi@nw.kanagawa-it.ac.jp

Toshiyasu MATSUSHIMA
School of Sci. and Eng.,
Waseda Univ., Tokyo, Japan.
e-mail: toshi@matsu.mgmt.waseda.ac.jp

Shigeichi HIRASAWA
School of Sci. and Eng.,
Waseda Univ., Tokyo, Japan.
e-mail: hirasawa@hirasa.mgmt.waseda.ac.jp

I. Introduction

Variable size list decoder(VLD) is first discussed in Forney [2], which selects more than one candidate messages if the message passes the given criterion. Though list decoder is seldom used for applications, it is well-known that the Generalized Viterbi algorithm(GVA)[1] employs the fixed size list decoder(FLD) in its mean process. In this paper, we newly propose the GVA which uses VLD, instead of FLD. Then the coding theorem is obtained.

II. Preliminaries

We denote a discrete memory less channels characterized by $P = \{P_{ij},\ j \in A, i \in B\}$, where $A = \{0, 1, \cdots, a-1\}$ is the input alphabet and $B = \{0, 1, \cdots, b-1\}$ is the output alphabet. We are concerned with a q-ary tree code, each branch of which is assigned with v input alphabet. A N information sequence of q-ary alphabet can be denoted $\mathbf{u}_i{}^N = u_{i,1}u_{i,2}\cdots\cdots u_{i,t}\cdots u_{i,N}$, where, $u_{i,t} \in \mathcal{U}$, $t = 1, 2, \cdots, N$, $i = 1, 2, \cdots, q^N$ and $\mathcal{U}$ is a finite alphabet of $\{0, 1, \cdots, q-1\}$. The path from the root to the N-th level of the tree specifies the information sequence $\mathbf{u}_i{}^N$ and its codeword sequence. So, the rate of the code is defined by $R = \frac{1}{v}\ln q$.

Let the sub-sequence of path $\mathbf{u}_i{}^N$ from the root to the n-th level be $\mathbf{u}_i{}^n$. Let the codeword sequence of the path $\mathbf{u}_i{}^N$ from the root to the n-th level be $\mathbf{x}_i^{vn}$, and the received sequence from the root to the n-th level be $\mathbf{y}^{vn}$, respectively.

III. Proposed algorithm

The algorithm is described as follows. Step 1~Step 2 are the same procedure of the original GVA, and Step 3 is for the decision rule for the variable size list decoder.

{ *Initial condition and Recursive procedure* }

(Step 1)*Initial condition* : At the level $n-1$, each state of q^{L-1} has their list, the set of survivors.

For the level n $(L \le n \le N)$, repeat (Step 2)~(Step 3), recursively.

(Step 2)*Path extension*: At the level n, all retained paths are extended by one branch as $\mathbf{u}^n = \mathbf{u}^{n-1}u$, $u \in \mathcal{U}$. Then each metric of Sq^L paths is calculated.

(Step 3)*Path selection and Testing*: At each state of q^{L-1}, we denote the likelihood of the k-th most path as $Pr(\mathbf{y}^{vn}|\mathbf{x}_{(k)}^{vn})$ [1]. Each path is testing with the following decision rule, if it should be listed or not.

For testing path $\mathbf{u}_m{}^N$,

$$\frac{Pr(\mathbf{y}^{vn}|\mathbf{x}_m^{vn})}{Pr(\mathbf{y}^{vn}|\mathbf{x}_{(1)}^{vn})} \le \Delta, \tag{1}$$

holds, $\mathbf{u}_m{}^N$ is retained in the list. Else, discarded.

{ *Final path selection at the check tail* }

By $L-1$ known symbols, q^{L-1} lists are reduced to one list with Step 2 ~Step 4. Then, by $T-(L-1)$ known symbols, the best path is selected among the survivors of the final list.

IV. Main results

Let the probability of decoding error be $Pr(E_1)$. It decreases as the threshold decreases. However, if one makes the threshold small recklessly, the number of retained survivors would be too large until the final selection at the check tail. Consequently, we have the definition for the error exponent for the proposed scheme as $\lim_{L\to\infty} -\frac{1}{vL}\ln Pr(E_1)$ with the taking the threshold Δ so that $-\frac{1}{vL}\ln Pr(E_2) \to 0$ $(L \to \infty)$, where $Pr(E_2)$ is the probability that more than one inncorrect path is listed at the final selection.

Theorem : There exists a tree code with the proposed scheme, whose exponent is lower bounded by $e_1^{new}(R)$,

$$e_1^{new}(R) = \max_{\mathbf{q},\sigma_1,\rho_1\in\mathcal{D}_1}\left\{E_o(\sigma_1,\rho_1,\mathbf{q}) + \sigma_1\cdot e_F(R)\right\},$$

$$E_o(\sigma_1,\rho_1,\mathbf{q})$$

$$= -\ln\left[\sum_{j\in B}\left(\sum_{i\in A} q_i P_{ji}^{1-\sigma_1}\right)\left(\sum_{k\in A} q_k P_{jk}^{\sigma_1/\rho_1}\right)^{\rho_1}\right],$$

$$\mathcal{D}_1 = \{0 \le \rho_1 \le 1,\quad \sigma_1 \ge 0, E_o(\sigma_1,\rho_1,\mathbf{q}) - \rho_1 R > 0\},$$

$$e_F(R) = \max_{\mathbf{q},\nu\in\mathcal{D}_3} E_{oF}(\nu,\mathbf{q}),$$

$$\mathcal{D}_3 = \{E_{oF}(\nu,\mathbf{q}) - \nu R > 0,\quad \nu > 0\},$$

$$E_{oF}(\nu,\mathbf{q}) = \sum_{k\in B}\sum_{j\in A} q_j P_{kj}\ln\left[\frac{P_{kj}^{1/\nu}}{\sum_{j\in A} q_j P_{kj}{}^{1/\nu}}\right]^{\nu}. \tag{2}$$

V. Conclusion

We proposed the new version of the GVA using VLD. Then the lower bound of its exponent is derived. At some channels, this bound is larger than that of using FLD. We also observed the property of the proposed scheme by computer simulations. It shows there exists some domains where the proposed scheme has lower error probability for the average retained path size, at the sacrifice of the retained path size fixed.

References

[1] T.Hashimoto, "A list-type reduced-constraint generalization of the Viterbi algorithm", *IEEE Trans. Inform.Theory*, vol.IT-33, no.6, pp.866-876, Nov.1987.

[2] G.D.Forney,Jr., "Exponential error bounds for erasure, list and decision feedback schemes", *IEEE Trans.Inf.Theory*, vol.IT-14, no.2, pp.206-220, Mar.1968.

[3] T.Hashimoto, "On the error exponent of convolutionaly coded ARQ", *IEEE Trans. Inform.Theory*, vol.IT-40, no.2, pp.567-575, Mar.1994.

[1] We denote the likelihood of the path $\mathbf{u}_k{}^N$ as $Pr(\mathbf{y}^{vn}|\mathbf{x}_k^{vn})$. On the other hand, we denote the likelihood of the k-th most path as $Pr(\mathbf{y}^{vn}|\mathbf{x}_{(k)}^{vn})$.