International Symposium on Information Theory and its Applications, ISITA2006
Seoul, Korea, October 29–November 1, 2006

Performance of Low-Density Parity-Check Codes for Burst Erasure Channels

Gou HOSOYA[†‡], Hideki YAGI[††], Toshiyasu MATSUSHIMA[†], and Shigeichi HIRASAWA[†]

[†] School of Science and Engineering, Waseda University, Okubo 3–4–1, Shinjuku-ku, Tokyo, 169–8555 Japan.

[††] Media Network Center, Waseda University, Totsuka-Machi 1–104, Shinjuku-ku, Tokyo, 169–8050 Japan.

[‡] E-mail: hosoya@hirasa.mgmt.waseda.ac.jp

Abstract

Performance of low-density parity-check (LDPC) codes with maximum likelihood decoding (MLD) for solid burst erasures is discussed. The columns of the parity-check matrix of LDPC codes are permuted to increase the distance between elements (DBEs) which are defined as a number of symbol positions between elements 1 at each row of the parity-check matrix. The column permutation method can change the burst erasure correction capabilities by both the sum-product decoding algorithm and MLD algorithm. We derive some properties and show from simulation results that large values of DBEs lead to good performance for MLD.

1. Introduction

The combination of LDPC codes with the sum-product (SP) decoding algorithm enables to have high performance with low decoding complexity [1]. Most of studies of LDPC codes assume random errors or random erasures. When we consider practical applications of LDPC codes, we must take into account correction capabilities of not only random errors or erasures but also burst ones. In order to adapt the code to burst channels, two approaches have been taken. The first approach is to improve decoding methods for the burst channels, and the second one is to construct or modify the codes suitable for the burst channels. The first one has been taken by A. W. Eckford et al. [7]. The second approach has been taken by the present authors [3], [8], M. Yang and W. E. Ryan [4] and T. Wadayama [5]. Yang et al. have proposed the $L_{\max}$ algorithm which can evaluate a length of a correctable maximum solid burst erasure for a given parity-check matrix of LDPC codes by an exhaustive search method. Wadayama has proposed a column permutation algorithm which can increase $L_{\max}$ for given LDPC codes. The works by Yang et al. and by Wadayama have considered only one solid burst erasure[1]. Present authors have also showed that for two or more solid burst erasures, the codes constructed by a column permutation based on increasing distance between elements (DBEs), which are a number of symbol positions between adjacent elements 1 at each row of the parity-check matrix, have good performance of erasure correction [8]. These approaches have assumed the SP decoding algorithm for erasure correction.

In this paper, the performance of low-density parity-check (LDPC) codes with maximum likelihood decoding (MLD) for solid burst erasures based on DBE is discussed. The column permutation method based on increasing DBE [8] can change the burst erasure correction capabilities not only by the SP decoding algorithm but also by MLD, while there is no degradation in the performance over random erasure channels. As a case of decoding over the erasure channel, the performance of the SP decoding algorithm and MLD are well relevant, since the SP decoding algorithm over the erasure channel can be recognized as a part of the efficient MLD algorithm [6] which utilizes the SP decoding at first and the Gaussian elimination next. We derive some properties and show from simulation results that large values of DBEs lead to good performance for MLD.

This paper is organized as follows. In Section 2, we describe LDPC codes and decoding for the erasure channel. In Section 3, we describe DBE and its some properties. In Section 4, we show some simulation results and discussions. Finally, concluding remarks are given in Section 5.

2. LDPC Codes and Erasure Correction

Let $H = [H_{mn}]$, $m \in [1, M]$, $n \in [1, N]$, be a parity-check matrix whose row and column lengths are M

[1] A "solid burst erasure" of length T stands for consecutive T erasures.

and N, respectively[2]. Let w_r and w_c be the row and the column weight of H, respectively. Let M be the number of rows of H which is given by $M = Nw_c/w_r$. In this paper, we consider binary regular (N, w_r, w_c) LDPC codes to simplify the discussion.

We assume a codeword $\boldsymbol{c} = (c_1, c_2, \ldots, c_N) \in \{0,1\}^N$ of the LDPC code of length N is transmitted through an erasure channel. $\boldsymbol{c}$ is disturbed by a sequence from the channel $\boldsymbol{e} = (e_1, e_2, \ldots, e_N) \in \{0, \epsilon\}^N$ where ϵ denotes an erasure, and the decoder receives a sequence $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{e}$. The addition of a binary symbol and the erasure symbol is defined as $0+\epsilon = \epsilon$, $1+\epsilon = \epsilon$. The decoder estimates the transmitted codeword from the received sequence.

Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be the index set of symbol positions. And let $\mathcal{E} \subseteq \mathcal{N}$ and $\bar{\mathcal{E}} = \mathcal{N} \setminus \mathcal{E}$ be the index sets of erased symbol positions and known symbol positions, respectively. From the definition of the parity-check matrix, we can write

$$\boldsymbol{c}H^{\mathrm{T}} = \boldsymbol{c}_{\mathcal{E}}H_{\mathcal{E}}^{\mathrm{T}} \oplus \boldsymbol{c}_{\bar{\mathcal{E}}}H_{\bar{\mathcal{E}}}^{\mathrm{T}} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{c}_{\mathcal{E}}$ and $H_{\mathcal{E}}$ are a subvector of $\boldsymbol{c}$ and a submatrix of $\boldsymbol{c}$ and H which consist of those columns indexed by $\mathcal{E}$, respectively. Since $\boldsymbol{c}_{\bar{\mathcal{E}}}H_{\bar{\mathcal{E}}}^{\mathrm{T}}$ is known to a decoder,

$$\boldsymbol{c}_{\mathcal{E}}H_{\mathcal{E}}^{\mathrm{T}} \quad = \quad \boldsymbol{c}_{\bar{\mathcal{E}}}H_{\bar{\mathcal{E}}}^{\mathrm{T}} = \boldsymbol{s}', \tag{2}$$

where $\boldsymbol{s}' = (s'_1, s'_2, \ldots, s'_M) \in \{0,1\}^M$ is a syndrome sequence calculated by $\boldsymbol{c}_{\bar{\mathcal{E}}}H_{\bar{\mathcal{E}}}^{\mathrm{T}}$. Maximum likelihood decoding (MLD) obtaines the erased (unknown) sequence $\boldsymbol{c}_{\mathcal{E}}$ from the simultaneous equations $\boldsymbol{c}_{\mathcal{E}}H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$. Therefore MLD can decode the received sequence correctly iff $rank[H_{\mathcal{E}}] = |\mathcal{E}|$ where $rank[A]$ denotes the rank of a matrix A. Note that the rank of $H_{\mathcal{E}}$ is given by an erasure pattern $\mathcal{E}$, so this is not equal to the rank of parity-check matrix H.

In [6], an efficient MLD algorithm for LDPC codes over the binary erasure channel has been proposed. The algorithm combines the SP decoding and the Gaussian elimination (GE). First it implements the SP decoding to correct some erased symbols, and next implements the GE to correct remaining erased symbols. From this algorithm, it can be seen that the performance of the SP decoding algorithm and MLD are well relevant since the SP decoding algorithm is a part of MLD.

3. Burst Erasure Correction Capability based on DBEs

3.1. Definitions of DBEs

We define the following set for $m \in [1, M]$.

$$\mathcal{A}(m) \triangleq \{n : H_{mn} = 1\} = \{n_{m,1}, n_{m,2}, \ldots, n_{m,w_r}\},$$

where $n_{m,1} < n_{m,2} < \ldots < n_{m,w_r}$.

We define the *distance between elements* (DBE) as the number of symbol positions between adjacent elements 1 at each row of the parity-check matrix.

Definition 1. [DBE] The DBEs $d_{m\gamma}$, $m \in [1, M]$, $\gamma \in [1, w_r - 1]$, the minimum value of DBEs $D_{\min}$, and the maximum value of DBEs $D_{\max}$ are defined by the following equations, respectively:

$$\begin{aligned} d_{m\gamma} &\triangleq n_{m,\gamma+1} - n_{m,\gamma}, && (3)\\ D_{\min} &\triangleq \min_{m,\gamma}\{d_{m\gamma}\}, && (4)\\ D_{\max} &\triangleq \max_{m,\gamma}\{d_{m\gamma}\}. && (5)\end{aligned}$$

Moreover, we define D_{left} (D_{right}) as the maximum (minimum) value of the leftmost (rightmost) symbol position of the element 1 for the parity-check matrix by the following equations, respectively:

$$\begin{aligned} D_{\mathrm{left}} &\triangleq \max_{m}\{n_{m,1}\}, && (6)\\ D_{\mathrm{right}} &\triangleq \max_{m}\{N - n_{m,w_r} + 1\}. && (7)\end{aligned}$$

□

3.2. Column Permuted Parity-Check Matrix [8]

For a parity-check matrix, the DBEs are changed by column permutation. Let D_{ave} be an arithmetic average value of DBEs defined as

$$D_{\mathrm{ave}} \triangleq \frac{1}{M(w_r - 1)} \sum_{m=1}^{M} \sum_{\gamma=1}^{w_r-1} d_{m\gamma}. \tag{8}$$

We state structures of the parity-check matrix that maximizes the value of D_{ave}. To increase DBEs, we consider the following two conditions: (i) D_{ave} has a large value, (ii) $D_{\min} > \delta$ where δ is some positive constant. From Eq. (3), we have

$$\sum_{\gamma=1}^{w_r-1} d_{m\gamma} = n_{m,w_r} - n_{m,1}. \tag{9}$$

We can easily see that D_{ave} depends on a difference of column positions between the leftmost element 1 and the rightmost element 1 at each row of a parity-check matrix since we have

$$D_{\mathrm{ave}} = \frac{1}{M(w_r - 1)} \sum_{m=1}^{M} \big(n_{m,w_r} - n_{m,1}\big). \tag{10}$$

[2]For two integers i and j ($i \leq j$), $[i, j]$ denotes the set of integers from i to j.

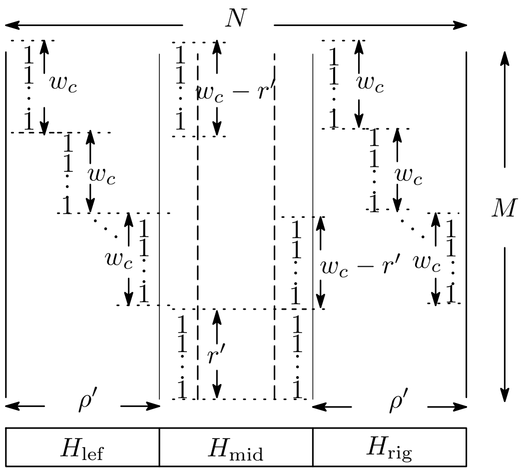

Figure 1: An example of H that has a form of Eq. (11)

Therefore the condition (i) implies that the sum of DBEs ($n_{m,w_r} - n_{m,1}$ for $\forall m$) has a large value.

We now let the leftmost (rightmost) symbol position at each row of H be as small (large) as possible. Let $r' \triangleq M \bmod w_c$ and $\rho' \triangleq \frac{M-r'}{w_c} = \frac{N}{w_r} - \frac{r'}{w_c}$. Assume that H has a following form:

$$H \triangleq [H_{\mathrm{lef}}, H_{\mathrm{mid}}, H_{\mathrm{rig}}]. \tag{11}$$

H_{lef} and H_{rig} are $M \times \rho'$ matrices such that the weights of r' rows and $M - r'$ rows of these matrices are 0 and 1, respectively, and the weights of columns of those are w_c. Fig. 1 shows an example of H that has the form of Eq. (11). H_{lef} constitutes the leftmost ρ' columns of H. Assume that column positions of element 1 at those columns are $n_{m,1}$, the leftmost element 1 at each row. Similarly, H_{rig} constitutes the rightmost ρ' columns of H. Assume again that column positions of element 1 at those columns are n_{m,w_r}, the rightmost element 1 at each row. Then the following lemma holds.

Lemma 1. The parity-check matrix H that has the form of Eq. (11) maximizes the right-hand side of Eq. (8). □

We show the following theorem on the relation of D_{ave} and (N, w_r, w_c) LDPC codes.

Theorem 1. Any parity-check matrix of (N, w_r, w_c) LDPC codes satisfies the following equation:

$$D_{\mathrm{ave}} \leq \frac{N}{w_r}. \tag{12}$$

□

Proof. See [8]. □

To modify the parity-check matrix of LDPC codes suitable for burst erasures, we permute the columns of a parity-check matrix of LDPC codes to have the form Eq. (11) as nearly same as possible. Detailed description of the column permutation algorithm is omitted. See [8] for details.

3.3. Properties

We assume that L solid burst erasures whose total length is T have been contained in a received sequence. From a point of view of the MLD for erasure channels, it is desirable that the rank of submatrix $H_{\mathcal{E}}$ has a large value. It is hard to see the relationship between the rank of $H_{\mathcal{E}}$ and DBEs, so we show some properties which imply the relationship between DBEs and the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$.

Theorem 2. Assume that a solid burst erasure of length V has been contained in a received sequence. The number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ is exactly $w_c V$ for $D_{\min} \geq V$, and is at least $D_{\min} \times w_c$ otherwise. □

Proof. It is straightforward and omitted. □

Let us denote the number of solid burst erasures by L. When $L = 1$ solid burst erasure has been occurred, Theorem 2 holds for $V = T$. When $L > 1$ solid burst erasures have been occurred, Theorem 2 holds for $V = \lceil \frac{T}{L} \rceil$ since the length of one of solid burst erasures is at least $V = \lceil \frac{T}{L} \rceil$.

From Theorem 2, to have the large number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$, we should have $D_{\min}$ a large value. Although the rank of $H_{\mathcal{E}}$ is not necessarily equal to the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}} = \boldsymbol{s}'$, a large minimum value of DBEs $D_{\min}$ leads to a large value of the rank of $H_{\mathcal{E}}$ in general.

Theorem 3. Assume that $L = 1$ solid burst erasure of the length T has been contained in a received sequence. The number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ is exactly M when T satisfies all the following equations:

$$T \geq D_{\mathrm{left}}, \tag{13}$$

$$T \geq D_{\mathrm{right}}, \tag{14}$$

$$T \geq D_{\max} + 1. \tag{15}$$

□

Proof. The proof is accomplished by showing all rows in $H_{\mathcal{E}}$ have at least one elements 1. Consider the three cases that $L = 1$ solid burst erasure of the length T has occurred at (i) the symbol position $n = 1$, (ii) the symbol position $n = N - T + 1$, and (iii) the symbol positions $n = 2, 3, \ldots, N - T$.

In the case (i), the set of erased symbol positions $\mathcal{E}$ satisfies $\mathcal{E} \in [1, T]$. If Eq. (13) holds, the leftmost elements 1 at the symbol position $n_{m,1}$ in all rows of H are contained in $H_{\mathcal{E}}$, then it implies that the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ is exactly M.

In the case (ii), the set of erased symbol positions $\mathcal{E}$ satisfies $\mathcal{E} \in [N - T + 1, N]$. If Eq. (14) holds, the rightmost elements 1 at the symbol position n_{m,w_r} in all rows of H are contained in $H_{\mathcal{E}}$, then it implies that the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ is exactly M.

In the case (iii), the sets of erased symbol positions $\mathcal{E}$ satisfies $\mathcal{E} \in [n, n + T - 1]$. If Eq. (15) holds, at least one element 1 at each row is in $H_{\mathcal{E}}$, then it implies that the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ is exactly M.

From the cases (i) $\sim$ (iii), Eqs. (13) $\sim$ (15) hold.

$\square$

From Theorem 3, it is better to make $D_{\max}$, D_{left}, and D_{right} small. It is expected that as the value of $D_{\min}$ be a large value, the value of $D_{\max}$ takes a small value in general since the maximum value of D_{ave} (the sum of DBEs) is upper bounded. It can be easily seen that the parity-check matrix of the form of Eq. (11) minimizes the values of D_{left} and D_{right} since the left-most (rightmost) position of element 1 at each row of the parity-check matrix of the form of Eq. (11) is as small (large) as possible.

4. Simulation Results and Discussion

In order to demonstrate MLD performance of codes, we show some simulation results.

4.1. Conditions for Simulations

We use LDPC codes of $N = 500$, $w_r = 6$, and $w_c = 3$ in simulations. We construct three codes (denoted by "Code 1", "Code 2", and "Code 3") by different seeds of random generator (these original codes are denoted by "Original"). For each code, we permute columns of a parity-check matrix of the original codes. The permutation methods are 1) based on DBEs [8] (denoted by "DBE"), 2) based on increasing $L_{\max}$ [5] (denoted by "$L_{\max}$")[3], and 3) to have values of many DBEs 1 (denoted by "Small"). We decode until at least 5×10^5 codewords are transmitted or 30 codewords are failed to decode by MLD. We denote the number of solid burst erasures by L and the total length of erasures by T.

[3]Note that this column permutation method may delete some columns for increasing $L_{\max}$ [5]. However, we do not delete any columns for a fair comparison.

Table 1 (A): The values of D_{ave} for each code

	Original	DBE	$L_{\max}$	Small
Code 1	62.4	82.4	63.5	54.1
Code 2	62.8	82.4	65.1	54.0
Code 3	61.2	82.3	62.5	53.5

Table 1 (B): The values of $D_{\max}$ ($D_{\min}$) for each code

	Original	DBE	$L_{\max}$	Small
Code 1	338 (1)	203 (53)	338 (1)	373 (1)
Code 2	394 (1)	202 (53)	394 (1)	360 (1)
Code 3	439 (1)	199 (53)	439 (1)	331 (1)

Table 1 (C): The values of D_{left} (D_{right}) for each code

	Original	DBE	$L_{\max}$	Small
Code 1	375 (409)	115 (135)	409 (375)	336 (494)
Code 2	400 (415)	115 (125)	399 (400)	339 (488)
Code 3	436 (445)	163 (135)	445 (436)	400 (494)

4.2. The Values of DBEs

We show the values D_{ave}, $D_{\max}$, $D_{\min}$, D_{left}, and D_{right} for each code in Tables 1 (A) $\sim$ 1 (C), respectively.

From Table 1 (A), the values of D_{ave} of the code "DBE" are the largest. Note that the upper bound on D_{ave} of these codes is $N/w_r \simeq 83.3$ [3], [8]. Since the code "DBE" is constructed to have a large value of DBEs, values of $D_{\min}$ of the code "DBE" are 53, and that of the other codes are all 1 from Table 1 (B). From Tables 1 (B) and 1 (C), the values of $D_{\max}$, D_{left}, and D_{right} of the code "DBE" are considerably smaller than those of the other codes.

4.3. Decoding for Several Burst Erasures

We assume L solid burst erasures whose total length is $T = 230$, have occurred. Lengths for each burst are chosen from a seed of random generator. Fig. 2 shows decoding performance for the total burst length $T = 230$. The horizontal axis shows the number of solid burst erasures L and the vertical axis shows the word error rate (WER).

From Fig. 2, the performance of the code "DBE" is better than that of the other codes. Note that when $L = 1$ and 2, the code "DBE" produces the zero WER.

Next, we assume that $L = 1$ and $L = 5$ solid burst erasures for a fixed total length T have occurred. Lengths for each burst are chosen from a seed of random generator. Figs. 3 (A) and 3 (B) show decoding performance for the number of solid bursts $L = 1$ and $L = 5$, respectively. The horizontal axis shows the total length of solid bursts T.

From the figures, the performance of the code

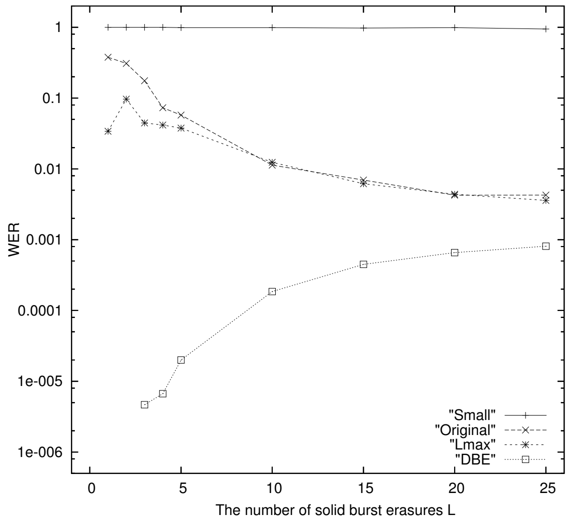

Figure 2: Decoding performance for L solid burst erasures whose total length $T = 230$. Note that when $L = 1$ and 2, the code "DBE" produces the zero WER.

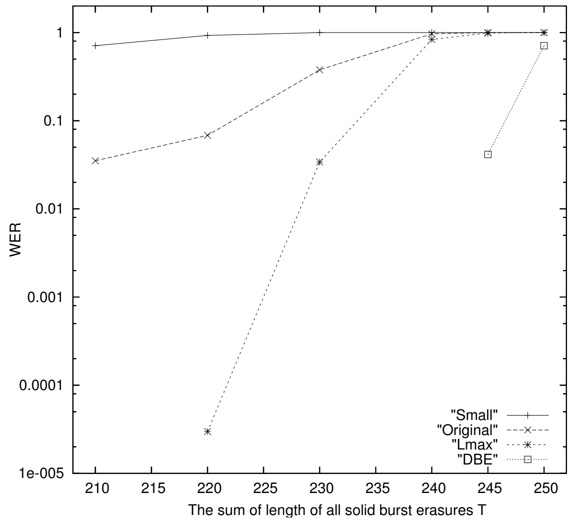

Figure 3 (A): Decoding performance for $L = 1$ solid burst erasures whose total length is T. Note that when $L = 1$ and $T = 210 \sim 240$, the code "DBE" produces zero WER.

"DBE" is better than that of the other codes. The difference of the performance between the code "DBE" and the other codes are large when T is small and becomes smaller as T becomes large. The WER of the code "DBE" is approximately 10^3 times smaller than that of the code "$L_{\max}$" when $T = 220$ in Fig. 3 (B).

4.4. Discussions

From simulation results in Section 4.3, the performance of the code "DBE" depends on the number of solid burst erasures L. When L is small, the difference of the performance between the code "DBE" and the other codes is large and becomes small as L becomes large.

The key idea of a column permutation method for the MLD is to make the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}} = \boldsymbol{s}'$ a large value. Tables 2 (A) $\sim$ 3 (B) show the average number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}} = \boldsymbol{s}'$ and the rank of $H_{\mathcal{E}}$ when $L = 1$ and 5, respectively.

Note: When $L = 1$ and $T = 210 \sim 240$ in Table 2 (B), the code "DBE" produces the zero WER, so the rank of $H_{\mathcal{E}}$ equals to the number of erasures T. When $L = 5$ and $T = 210 \sim 230$ in Table 3 (B), the code "DBE" also produces the zero WER, so the rank of $H_{\mathcal{E}}$ equals to the number of erasures T. □

From Tables 2 (A) and 3 (A), the average number of equations of the code "DBE" is larger than the other codes and those of the codes "Original" and "$L_{\max}$" are almost the same. If the number of equations is small, then it implies that the rank in $H_{\mathcal{E}}$ tneds to be small. From Tables 2 (B) and 3 (B), we can see that the rank in $H_{\mathcal{E}}$ of the code "DBE" is larger than those of the other codes. When $T = 230$ and $L = 5$ from Table 3 (B), since $D_{\min}$ of the codes "DBE" are all 53 and there is at least one solid burst erasure of the length at least 46, it is guaranteed from Theorem 2 that the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ is at least 138. However, $D_{\min}$ of the other codes are all 1, so it is only guaranteed that the number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}} = \boldsymbol{s}'$ is at least 3.

5. Concluding Remarks

In this paper, we show the correction capabilities of LDPC codes for solid burst erasures decoded by MLD based on DBEs. From simulation results, the codes with large values of DBEs have a good performance when L is small. We also show that the performance of the code with small values of DBEs is bad. We show from Theorems 2 and 3 that large values of $D_{\min}$ and small values of $D_{\max}$, D_{left}, and D_{right} leads to the large number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$. We also show from simulation results that the rank of $H_{\mathcal{E}}$ and the average number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ of the code "DBE" are larger than those of the other codes. Since performances of the MLD relate to the rank of $H_{\mathcal{E}}$, larger the average number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}}^{\mathrm{T}} = \boldsymbol{s}'$ for each code, the smaller WER of simulation results can be obtained.

Theoretical analyses of the performance between the DBEs and the rank of $H_{\mathcal{E}}$ is remained for further investigation.

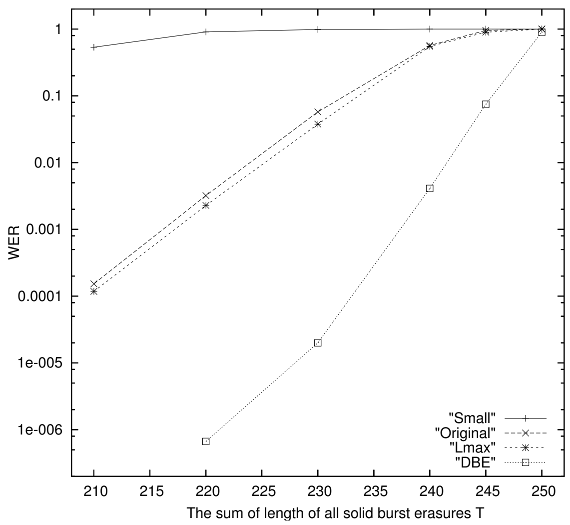

Figure 3 (B): Decoding performance for $L = 5$ solid burst erasures whose total length is T

Acknowledgements

This work is partially supported by Waseda University Grant for Special Research Project No. 2005A-952 and No. 2006B-293.

Table 2 (A): The average number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}} = \boldsymbol{s}'$ when $L = 1$

T	Original	DBE	$L_{\max}$	Small
210	230.56	250.00	234.62	209.59
220	233.18	250.00	236.70	213.43
230	235.51	250.00	238.61	215.94
240	237.01	250.00	240.02	218.90
245	238.07	250.00	240.77	220.22
250	238.91	250.00	241.26	221.56

Table 2 (B): The rank of $H_{\mathcal{E}}$ when $L = 1$

T	Original	DBE	$L_{\max}$	Small
210	206.77	210.00	210.00	105.76
220	210.49	220.00	220.00	68.95
230	184.12	230.00	226.81	42.20
240	74.34	240.00	110.06	36.32
245	48.42	239.78	56.84	33.22
250	40.74	140.68	42.17	31.38

Table 3 (A): The average number of equations in $\boldsymbol{c}_{\mathcal{E}} H_{\mathcal{E}} = \boldsymbol{s}'$ when $L = 5$

T	Original	DBE	$L_{\max}$	Small
210	237.61	247.95	238.04	212.60
220	239.50	248.57	239.92	216.87
230	240.96	249.02	241.57	219.65
240	242.19	249.34	242.51	222.42
245	242.60	249.49	242.99	223.53
250	243.37	249.63	243.58	224.81

Table 3 (B): The rank of $H_{\mathcal{E}}$ when $L = 5$

T	Original	DBE	$L_{\max}$	Small
210	209.99	210.00	209.99	141.50
220	219.70	220.00	219.79	78.91
230	224.10	230.00	226.18	51.30
240	155.87	239.57	161.29	40.90
245	70.24	236.34	89.49	38.34
250	40.44	115.59	44.57	34.37

References

[1] R. G. Gallager, "Low density parity check codes," *IRE Trans. Inform. Theory*, vol. 8, pp. 21–28, Jan. 1962.

[2] C. Di, D. Proietti, I. E. Telatar, T. J. Richardson, and R. L. Urbanke, "Finite-length analysis of low-density parity-check codes on the binary erasure channel," *IEEE Trans. Inform. Theory*, vol. 48, no. 6, pp. 1570–1579, June 2002.

[3] G. Hosoya, H. Yagi, M. Kobayashi, and S. Hirasawa, "Construction methods of low-density parity-check codes for burst error channels," (in Japanese) *IEICE Technical Report*, vol. 104, no. 302, pp. 61–66, July 2003.

[4] M. Yang and W. E. Ryan, "Performance of efficiently encordable low-density parity-check codes in noise bursts on the EPR4 channel," *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 507–512, March 2004.

[5] T. Wadayama, "Greedy construction of LDPC codes for burst erasure channel," *IEICE Technical Report*, vol. 104, no. 302, pp. 35–40, Sept. 2004.

[6] D. Burshtein and G. Miller, "An efficient maximum-likelihood decoding of LDPC codes over the binary erasure channel," *IEEE Trans. Inform. Theory*, vol. 50, no. 11, pp. 2837–2844, Nov. 2004.

[7] A. W. Eckford, F. R. Kschischang, and S. Pasupathy, "Analysis of low-density parity-check codes for the Gilbert-Elliott channel," *IEEE Trans. Inform. Theory*, vol. 51, no. 11, pp. 3872–3889, Nov. 2005.

[8] G. Hosoya, H. Yagi, T. Matsushima, and S. Hirasawa, "A modification method for constructing low-density parity-check codes for burst erasures," *to appear in IEICE Trans. Fundamentals.*